Michigan Assessment Consortium

Learning Point

Disciplinary literacy in mathematics and the need to develop mathematical reasoning

Disciplinary literacy encompasses the ways of being, inquiring, comprehending, and communicating of an insider within a specialized community. Mathematical literacy depends on developing mathematical reasoning–the ability to use what a student already knows to develop solutions for what they do not *yet know* (IES 2025). This is the essence of the work mathematicians do, yet it is almost absent from the current K-12 mathematics classroom experience. This Learning Point suggests that the focus in math classes on algorithms as *answer-finding tools* results in a dearth of the mathematical reasoning ability that defines a mathematician.

> ***"Algorithms trap students from progressing."***

Teaching algorithms vs. developing mathematical reasoning

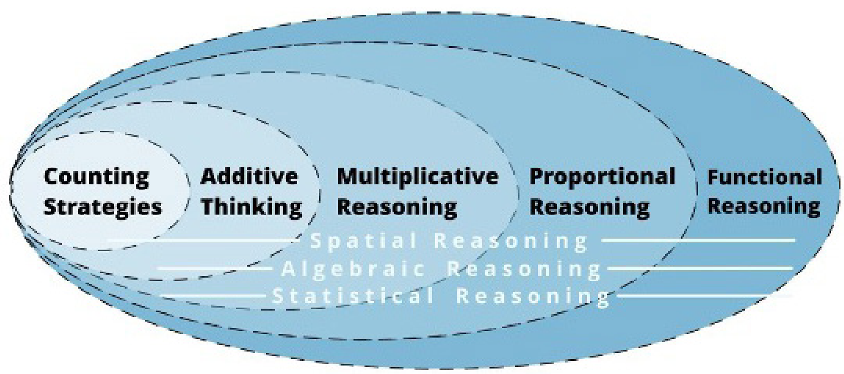
The current outsized emphasis on teaching algorithms, which function as a way for students to get answers without understanding either the process they are using or the answers they get, causes most students to graduate high school with a distorted view of what mathematics even is.

Mathematical reasoning is not the shallow ability to find the answer to a specific question (which any calculator can perform faster and more accurately). It is rather the ability to take previously developed reasoning and apply it in a new theater.

Mathematical literacy requires focus on Major Strategies

Mathematical reasoning is best developed through a focus on the Major Strategies (Harris, 2025). The finite list of mathematical problem-solving approaches shown in Figure 1 defines all the methods of reasoning a student needs access to in order to progress from counting, to addition and subtraction, to multiplication and division, to proportions, and finally to functions.

Figure 1. The Development of Mathematical Reasoning



This list of major relationships that lead to strategies consist of ideas like:

- Getting to a friendly number
- Over estimating and adjusting
- Creating an equivalent problem that's easier to solve

Teachers of mathematical literacy require a baseline level of mathematical reasoning. Not through any fault of their own, this is often absent. It plays out as a multi-generation bad game of telephone, with each generation resorting to more and more tricks to get answers because they cannot rely on the mathematical reasoning they were never taught.

Algorithms are amazing human achievements and powerful and useful in their sphere as general solutions for computers to use. However, mathematicians themselves almost never use algorithms to get answers to the one-off kinds of problems in textbooks (Dowker, 1992). In the K-12 mathematics classroom algorithms are inhibitive, terrible teaching tools. True, some learners can transcend those limitations, but even they would do better with intentional instruction in mathematical reasoning.

Algorithms create a digit-oriented trap

Algorithms trap students from progressing in three general ways, one of which is the digit-oriented trap. This trap means that the traditional algorithms trap students into viewing numbers as lists of digits instead of considering and operating on the magnitudes (sizes) of the numbers.

For example, 99 x 27. Students could develop multiplicative reasoning by considering that 99 27s is really close to 100 27s, and reasoning from there that you'll need one less 27 to find the solution (2700 - 27 = 2673). Instantly we have a good approximation, and students' brains get the mental exercise of dealing with the actual values in the problem.

However, the traditional algorithm has students consider the digits 9, 9, 2, 7. Then students:

- perform several single-digit multiplications,
- write down those answers as digits,
- add the rows in columns of digits, and
- treat every number in those steps as digits, digits, digits.

Additionally, digits are used in the least intuitive *order*. If a student is thinking about 99 27s or 27 99s, they are reasoning about almost 100 27s or they might think about a little less than 30 99s. They never develop a sense of the "-ish" answer, a sense of reasonableness (Boaler, 2024). But the traditional algorithm starts with 7 x 9, the smallest and least consequential numbers in the problem. This works against students' intuition, sending the message, "Don't think in math class; do steps whether they make sense or not. Math is about mimicking, not reasoning." This thinking is the opposite of mathematical literacy.

> ***"The processes of learning and figuring used by those currently successful in mathematics, the essence of mathematical literacy, can be taught to every student."***

A better way to mathematical literacy

Problem Strings like the one shown in Figure 2 help give students the necessary high doses of mathematical patterns so they can reason appropriately. The teacher gives the class each problem one at a time, then represents student thinking using a mathematical model, crafts conversations about the important mathematics, and helps students draw important conclusions.

Students talk about how 9 groups is just one group less than 10, 99 groups is one less group than 100, and 50 groups is just half of 100 groups. They are now primed to generalize reasoning about 49 groups, just one less than 50.

In this way, the development of mathematical reasoning becomes a question of equity. The current approaches work—barely—only for the most advantaged students— those who possess the elusive, ill defined, and deficit-thinking rooted "math gene." The reality is that the processes of learning and figuring used by those currently successful in mathematics, the essence of mathematical literacy, can be taught to every student.

Figure 2.

10 x 27	**If one pack of gum has 27 sticks, how many sticks are in 10 packs?**
9 x 27	**How many sticks are in 9 packs? Did anyone use the 10 packs to help?**
100 x 27	**How many sticks are in 100 packs?**
99 x 27	**How many in 99 packs? How do you know? Did you use 100 packs? Could you?**
50 x 27	**How many in 50 packs? Just half of 100 packs?**
49 x 27	**How about just 49 packs? How could you use the 50 packs?**
How could you reason about finding 49 packs no matter how many sticks are in those packs?	

References

Boaler, Jo. 2024. *Math-Ish: Finding Creativity, Diversity, and Meaning in Mathematics*. HarperOne, an imprint of HarperCollins Publishers.

Dowker, A. 1992. "Computational estimation strategies of professional mathematicians." *Journal for Research in Mathematics Education*, 23(1), 44–55. National Council of Teachers of Mathematics

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Institute of Education Sciences (IES). 2024. Development of mathematical reasoning. Retrieved Jan 27, 2025, from https://ies.ed.gov/ncee/edlabs/infographics/pdf/REL_SE_Development_of_Mathematical_Reasoning.pdf.

TO LEARN MORE

Developing Mathematical Reasoning: Avoiding the Trap of Algorithms, by Pamela Harris. Corwin, 2025.

www.corwin.com/books/dmr-289132

The Development of Mathematical Reasoning (blog)

www.mathisfigureoutable.com/blog/development